Effective resistivity of magnetic multilayers

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In heterogeneous system, the correspondence between calculated and measured quantities, such as the conductivity or the resistivity, is not obvious since the former ones are local quantities whereas the latter ones are often average values over the sample. In this report, we show explicitly how the correspondence can be done in the case of magnetic multilayers.

In the linear response regime, the electric current $\mathbf{J}$ at position $\mathbf{r}$ is related to the electric field $\mathbf{E}$ at position $\mathbf{r}'$ through[1]

$$J_i(\mathbf{r}) = \int_{-\infty}^{+\infty} d\mathbf{r}' \sum_j \sigma_{ij}(\mathbf{r},\mathbf{r}')E_j(\mathbf{r}'), \tag{1}$$

where i and j refer to the space directions $\{x,y,z\}$. The conductivity σ_{ij} depends of both $\mathbf{r}$ and $\mathbf{r}'$ (two-points conductivity), it is a local quantity which is expressed in the Kubo formalism[2] as a current-current correlation function

$$\sigma_{ij}(\mathbf{r},\mathbf{r}') = \lim_{\omega\to 0}\frac{1}{\omega}\int_0^{+\infty} dt\, e^{i\omega t}\,\langle\, [\, \mathbf{j}(\mathbf{r},t)\ ;\ \mathbf{j}(\mathbf{r}',0)\,]\,\rangle\,, \tag{2}$$

where $\mathbf{j}$ is the current density and the bracket $[\,A\ ;\ B\,] = AB - BA$. The notation $\langle\cdots\rangle$ refers to the configurational average. The question we address in this report is how to link the calculated quantities given by Eq. (2) which are local (position dependent) with the measured quantities which are mostly non-local. Indeed, the size of the electric contacts used to measure the conductivity (by measuring the current and the voltage) are generally quite large (from μm to mm). Then the measured quantity is a kind of average over a part of the sample. Strictly speaking, it is rather an effective value than an average value. The relation between the measured and calculated quantities varies strongly with the geometry of the system.

Let us start with a three-dimensional homogeneous system in the stationary regime. In this case, the electric current and electric field do not dependent of the position, thus Eq. (1) reduces to $J_i = \sum_j \bar{\sigma}_{ij}E_j$ where we have introduced the effective conductivity

$$\bar{\sigma}_{ij} \equiv \int_{-\infty}^{+\infty} d\mathbf{r}'\ \sigma_{ij}(\mathbf{r},\mathbf{r}'), \tag{3}$$

which is nothing else than the average value over the sample. The $\mathbf{r}$-dependence in the r.s.h. of Eq. (3) is irrelevant since the system is invariant by translation. Then, for an homogeneous system, the calculated (two-points conductivity $\sigma_{ij}(\mathbf{r},\mathbf{r}')$) and the measured (effective conductivity $\bar{\sigma}_{ij}$) quantities are related by a simple relation.

It is not more the case for heterogeneous systems because the electric current and electric field are not uniform through the sample. In this report, we treat the case of a multilayer formed by a superposition of identical cells which have the dimension $\{a_0, a_0, La_0\}$ where a_0 is the interatomic distance and L the number of layers in the cell (see Fig. (1a)). We have still an invariance by translation in each direction but the periodicity is different in comparison to the three-dimensional homogeneous system. The indication $\mathbf{r}$ of the position of one site must be replaced by the indication $\mathbf{R}$ of the position of the cell plus the indication l of the position of the site in the cell (l varies from 1 to L). For a multilayer, it is more appropriate to write Eq. (1) under the form

$$J_i^{l_1}(\mathbf{R}) = \int_{-\infty}^{+\infty} d\mathbf{R}' \sum_{j,l_2} \sigma_{ij}^{l_1 l_2}(\mathbf{R},\mathbf{R}')E_j^{l_2}(\mathbf{R}'), \tag{4}$$

because we have a translational invariance with respect to the vectors $\mathbf{R}$ and $\mathbf{R}'$, thus the electric current and electric field do not depend of the cell position. Following the same procedure than before, we write

$$J_i^{l_1} = \sum_{j,l_2} \sigma_{ij}^{l_1 l_2} E_j^{l_2}, \tag{5}$$

where

$$\sigma_{ij}^{l_1 l_2} \equiv \int_{-\infty}^{+\infty} d\mathbf{R}'\ \sigma_{ij}^{l_1 l_2}(\mathbf{R},\mathbf{R}'). \tag{6}$$

However, we can not design this quantity as the effective conductivity since it is a local quantity through the l_1 and l_2 indices. Since a multilayer is formed by a succession of layers made of different material, it is not possible to assume that the electric current and electric field are layer independent. To get the effective conductivity, we have to go further. In the literature, only approximate expressions have been proposed[3–5]. They are controlled by the relative value of two lengths: the mean-free-path λ and the average thickness d of the uniform layers in the cell. When $\lambda \gg d$ (homogeneous limit), the electrons can go through many layers without experience any scattering. They do not feel the difference between the layers and thus the cell can be considered as an homogeneous system. The layer dependence of the conductivity is not relevant and the multilayer can be treated as a three-dimensional homogeneous system (see Eq. (3)). When $\lambda \ll d$ (local limit), the electrons are scattered many times before they go out of one layer: the current in

one layer depends only weakly of the electric field in the other layers. As a consequence, the contributions of the two-points conductivity $\sigma_{ij}^{l_1 \neq l_2}$ are negligible, and Eq. (5) reduces to $J_i^l = \sum_j \sigma_{ij}^{ll} E_j^l$ which is still layer dependent. Thus, in the local limit, the effective conductivity can not be directly obtained. A general derivation of the exact effective conductivity valid in any regimes would be more appropriate. In this report, we present such a derivation. The starting point is Eq. (5) where the two-points conductivity $\sigma_{ij}^{l_1 l_2}$ links the local electric current $J_i^{l_1}$ to the local electric field $E_j^{l_2}$. The conductivity is a $(3L \times 3L)$ matrix. By inversion of this matrix, we get the matrix elements $\rho_{ij}^{l_1 l_2}$ (two-points resistivity) which link the local electric field $E_i^{l_1}$ to the local electric current $J_j^{l_2}$

$$E_i^{l_1} = \sum_{j,l_2} \rho_{ij}^{l_1 l_2} J_j^{l_2}. \tag{7}$$

The important thing now is to consider the experiments, in particular the geometry of the system and the way how the contacts are made. We distinguish two different geometries : the CIP geometry (current in the plane of the layers, see Fig. (1b)) and the CPP geometry (current perpendicular to the plane of the layers, see Fig. (1c)).

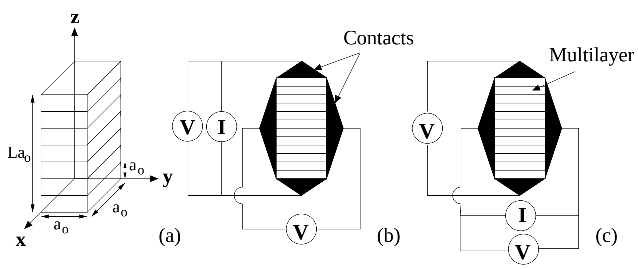

FIG. 1. (a) Schema of the unit cell for a multilayer; (b) CIP geometry; (c) CPP geometry.

In CPP geometry, the current is perpendicular to the layers. Due to current conservation, the perpendicular current is uniform through the cell (i.e., $J_z^l = J_z$) and the measured value is J_z. In CIP geometry, the current is parallel to the layers. Since the layers are different, the in-plane currents J_x^l and J_y^l are layer dependent. As it is shown in Fig (1c), the contacts cover several layers. We can assume that they cover at least a whole cell and that they do not have any influence on it, then what is measured are the average currents $\overline{J}_x \equiv \sum_l J_x^l/L$ and $\overline{J}_y \equiv \sum_l J_y^l/L$. For both geometries, the electrical field can have parallel and perpendicular components. Since the layers are different, the three components of the electric field should be layer dependent. However, the Maxwell equation in the stationary regime ($\nabla \times \mathbf{E} = 0$) combined with the fact that the electric field component E_z^l is uniform in the xy-plane impose that in-plane components of $\mathbf{E}$ are layer independent ($E_x^l = E_x$ and $E_y^l = E_y$). Thus, E_x and E_y correspond to the measured quantities. On the contrary, E_z^l stays layer dependent and what is measured is the average electric field $\overline{E}_z \equiv \sum_l E_z^l/L$.

By means of some transformations on Eqs. (5) and (7), we express J_x^l, J_y^l and E_z^l with the help of the uniform quantities E_x, E_y and J_z. For the moment, all the indices are kept. Eq. (5) can be written as

$$\sum_{l_2} \sigma_{zz}^{l_1 l_2} E_z^{l_2} = -\sum_{l_2} \sigma_{zx}^{l_1 l_2} E_x^{l_2} - \sum_{l_2} \sigma_{zy}^{l_1 l_2} E_y^{l_2} + J_z^{l_1}. \tag{8}$$

¿From Eq. (7), we have

$$\sum_{l_2} \rho_{xx}^{l_1 l_2} J_x^{l_2} + \sum_{l_2} \rho_{xy}^{l_1 l_2} J_y^{l_2} = E_x^{l_1} - \sum_{l_2} \rho_{xz}^{l_1 l_2} J_z^{l_2}, \tag{9}$$

and

$$\sum_{l_2} \rho_{yx}^{l_1 l_2} J_x^{l_2} + \sum_{l_2} \rho_{yy}^{l_1 l_2} J_y^{l_2} = E_y^{l_1} - \sum_{l_2} \rho_{yz}^{l_1 l_2} J_z^{l_2}. \tag{10}$$

These three equations can be written under the matrix form

$$\begin{pmatrix} \tilde{\rho}_{xx} & \tilde{\rho}_{xy} & 0 \\ \tilde{\rho}_{yx} & \tilde{\rho}_{yy} & 0 \\ 0 & 0 & \tilde{\sigma}_{zz} \end{pmatrix} \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{E}_z \end{pmatrix} = \begin{pmatrix} \tilde{I} & 0 & -\tilde{\rho}_{xz} \\ 0 & \tilde{I} & -\tilde{\rho}_{yz} \\ -\tilde{\sigma}_{zx} & -\tilde{\sigma}_{zy} & \tilde{I} \end{pmatrix} \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{J}_z \end{pmatrix}, \tag{11}$$

where $\tilde{I}$ is the $(L \times L)$ identity matrix and where we have introduced the following notations: the L-components vector $\tilde{A}_i$ ($A = J$ or E) and the $(L \times L)$ matrix $\tilde{a}_{ij}$ ($a = \sigma$ or ρ) define as

$$\tilde{A}_i \equiv \begin{pmatrix} A_i^1 \\ \vdots \\ A_i^L \end{pmatrix}, \qquad \tilde{a}_{ij} \equiv \begin{pmatrix} a_{ij}^{11} & \dots & a_{ij}^{1L} \\ \vdots & & \vdots \\ a_{ij}^{L1} & \dots & a_{ij}^{LL} \end{pmatrix}. \tag{12}$$

From Eq. (11), we get

$$\begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{E}_z \end{pmatrix} = \begin{pmatrix} \tilde{\rho}_{xx} & \tilde{\rho}_{xy} & 0 \\ \tilde{\rho}_{yx} & \tilde{\rho}_{yy} & 0 \\ 0 & 0 & \tilde{\sigma}_{zz} \end{pmatrix}^{-1} \times \begin{pmatrix} \tilde{I} & 0 & -\tilde{\rho}_{xz} \\ 0 & \tilde{I} & -\tilde{\rho}_{yz} \\ -\tilde{\sigma}_{zx} & -\tilde{\sigma}_{zy} & \tilde{I} \end{pmatrix} \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{J}_z \end{pmatrix}. \tag{13}$$

We inverse the first matrix in the r.s.h (it can be done numerically) and perform the multiplication of the two matrices, thus we get

$$\begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{E}_z \end{pmatrix} = \begin{pmatrix} \tilde{s}_{xx} & \tilde{s}_{xy} & -(\tilde{s}_{xx}\tilde{\rho}_{xz} + \tilde{s}_{xy}\tilde{\rho}_{yz}) \\ \tilde{s}_{yx} & \tilde{s}_{yy} & -(\tilde{s}_{yx}\tilde{\rho}_{xz} + \tilde{s}_{yy}\tilde{\rho}_{yz}) \\ -\tilde{p}_{zz}\tilde{\sigma}_{zx} & -\tilde{p}_{zz}\tilde{\sigma}_{zy} & \tilde{p}_{zz} \end{pmatrix} \times \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{J}_z \end{pmatrix}, \tag{14}$$

where $\tilde{s}_{ij}$ and $\tilde{p}_{zz}$ are $(L \times L)$ matrices defined through

$$\begin{pmatrix} \tilde{s}_{xx} & \tilde{s}_{xy} & 0 \\ \tilde{s}_{yx} & \tilde{s}_{yy} & 0 \\ 0 & 0 & \tilde{p}_{zz} \end{pmatrix} \equiv \begin{pmatrix} \tilde{\rho}_{xx} & \tilde{\rho}_{xy} & 0 \\ \tilde{\rho}_{yx} & \tilde{\rho}_{yy} & 0 \\ 0 & 0 & \tilde{\sigma}_{zz} \end{pmatrix}^{-1} . \quad (15)$$

By using the fact that E_x, E_y and J_z are layer-independent, we can sum over the columns and reduce the initial size $(3L \times 3L)$ of the matrix which appears in Eq. (14) to the size $(3L \times 3)$

$$\begin{pmatrix} J_x^1 \\ \vdots \\ J_x^L \\ J_y^1 \\ \vdots \\ J_y^L \\ E_z^1 \\ \vdots \\ E_z^L \end{pmatrix} = \begin{pmatrix} \sum_{l_2} s_{xx}^{1l_2} & \sum_{l_2} s_{xy}^{1l_2} & -\sum_{l_2,l_3} \left(s_{xx}^{1l_3}\rho_{xz}^{l_3l_2} + s_{xy}^{1l_3}\rho_{yz}^{l_3l_2}\right) \\ \vdots & \vdots & \vdots \\ \sum_{l_2} s_{xx}^{Ll_2} & \sum_{l_2} s_{xy}^{Ll_2} & -\sum_{l_2,l_3} \left(s_{xx}^{Ll_3}\rho_{xz}^{l_3l_2} + s_{xy}^{Ll_3}\rho_{xy}^{l_3l_2}\right) \\ \sum_{l_2} s_{yx}^{1l_2} & \sum_{l_2} s_{yy}^{1l_2} & -\sum_{l_2,l_3} \left(s_{yx}^{1l_3}\rho_{xz}^{l_3l_2} + s_{yy}^{1l_3}\rho_{yz}^{l_3l_2}\right) \\ \vdots & \vdots & \vdots \\ \sum_{l_2} s_{yx}^{Ll_2} & \sum_{l_2} s_{yy}^{Ll_2} & -\sum_{l_2,l_3} \left(s_{yx}^{Ll_3}\rho_{xz}^{l_3l_2} + s_{yy}^{Ll_3}\rho_{yz}^{l_3l_2}\right) \\ -\sum_{l_2,l_3} p_{zz}^{1l_3}\sigma_{zx}^{l_3l_2} & -\sum_{l_2,l_3} p_{zz}^{1l_3}\sigma_{zy}^{l_3l_2} & \sum_{l_2} p_{zz}^{1l_2} \\ \vdots & \vdots & \vdots \\ -\sum_{l_2,l_3} p_{zz}^{Ll_3}\sigma_{zx}^{l_3l_2} & -\sum_{l_2,l_3} p_{zz}^{Ll_3}\sigma_{zy}^{l_3l_2} & \sum_{l_2} p_{zz}^{Ll_2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ J_z \end{pmatrix} . \quad (16)$$

As we are interested by the average currents $\overline{J}_x \equiv \sum_l J_x^l/L$, $\overline{J}_y \equiv \sum_l J_y^l/L$ and the average electric field $\overline{E}_z \equiv \sum_l E_z^l/L$ (it is what we can get experimentally in such a system), we reduce the $(3L \times 3)$ matrix which appears in Eq. (16) to a (3×3) matrix

$$\begin{pmatrix} \overline{J}_x \\ \overline{J}_y \\ \overline{E}_z \end{pmatrix} = \begin{pmatrix} \|s_{xx}\| & \|s_{xy}\| & -\|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\| \\ \|s_{yx}\| & \|s_{yy}\| & -\|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\| \\ -\|p_{zz}\sigma_{zx}\| & -\|p_{zz}\sigma_{zy}\| & \|p_{zz}\| \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ J_z \end{pmatrix} , \quad (17)$$

where we have introduced the definition $\|a_{ij}\| \equiv \sum_{l_1,l_2} a_{ij}^{l_1l_2}/L$ in order to simplify the notations. This system of equations can be written under the form

$$\begin{pmatrix} 1 & 0 & \|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\| \\ 0 & 1 & \|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\| \\ 0 & 0 & -\|p_{zz}\| \end{pmatrix} \begin{pmatrix} \overline{J}_x \\ \overline{J}_y \\ J_z \end{pmatrix} = \begin{pmatrix} \|s_{xx}\| & \|s_{xy}\| & 0 \\ \|s_{yx}\| & \|s_{yy}\| & 0 \\ -\|p_{zz}\sigma_{zx}\| & -\|p_{zz}\sigma_{zy}\| & -1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ \overline{E}_z \end{pmatrix} . \quad (18)$$

From Eq. (18), the effective conductivity tensor $\tilde{\sigma}$ is

$$\overline{\sigma} = \begin{pmatrix} 1 & 0 & \|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\| \\ 0 & 1 & \|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\| \\ 0 & 0 & -\|p_{zz}\| \end{pmatrix}^{-1} \times \begin{pmatrix} \|s_{xx}\| & \|s_{xy}\| & 0 \\ \|s_{yx}\| & \|s_{yy}\| & 0 \\ -\|p_{zz}\sigma_{zx}\| & -\|p_{zz}\sigma_{zy}\| & -1 \end{pmatrix} . \quad (19)$$

The inversion of the first matrix and the product of the two matrices lead to the following expressions of the matrix elements for the effective conductivity

$$\overline{\sigma}_{xx} = \|s_{xx}\| - \frac{\|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\|\|p_{zz}\sigma_{zx}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{xy} = \|s_{xy}\| - \frac{\|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\|\|p_{zz}\sigma_{zy}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{xz} = -\frac{\|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{yx} = \|s_{yx}\| - \frac{\|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\|\|p_{zz}\sigma_{zx}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{yy} = \|s_{yy}\| - \frac{\|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\|\|p_{zz}\sigma_{zy}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{yz} = -\frac{\|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{zx} = \frac{\|p_{zz}\sigma_{zx}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{zy} = \frac{\|p_{zz}\sigma_{zy}\|}{\|p_{zz}\|},$$
$$\overline{\sigma}_{zz} = \frac{1}{\|p_{zz}\|}. \quad (20)$$

From Eq. (18), the effective resistivity tensor $\overline{\rho}$ is

$$\overline{\rho} = \begin{pmatrix} \|s_{xx}\| & \|s_{xy}\| & 0 \\ \|s_{yx}\| & \|s_{yy}\| & 0 \\ -\|p_{zz}\sigma_{zx}\| & -\|p_{zz}\sigma_{zy}\| & -1 \end{pmatrix}^{-1}$$

$$\times \begin{pmatrix} 1 & 0 & \|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\| \\ 0 & 1 & \|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\| \\ 0 & 0 & -\|p_{zz}\| \end{pmatrix}. \quad (21)$$

The inversion of the first matrix and the product of the two matrices lead to the following expressions of the matrix elements for the effective resistivity

$$\begin{aligned}
\overline{\rho}_{xx} &= \frac{1}{D}\|s_{yy}\|, \\
\overline{\rho}_{xy} &= -\frac{1}{D}\|s_{xy}\|, \\
\overline{\rho}_{xz} &= \frac{1}{D}\left(\|s_{yy}\|\|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\| \right. \\
&\quad \left. - \|s_{xy}\|\|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\|\right), \\
\overline{\rho}_{yx} &= -\frac{1}{D}\|s_{yx}\|, \\
\overline{\rho}_{yy} &= \frac{1}{D}\|s_{xx}\|, \\
\overline{\rho}_{yz} &= \frac{1}{D}\left(\|s_{xx}\|\|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\| \right. \\
&\quad \left. - \|s_{yx}\|\|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\|\right), \\
\overline{\rho}_{zx} &= \frac{1}{D}\left(\|s_{yx}\|\|p_{zz}\sigma_{zy}\| - \|s_{yy}\|\|p_{zz}\sigma_{zx}\|\right), \\
\overline{\rho}_{zy} &= \frac{1}{D}\left(\|s_{xy}\|\|p_{zz}\sigma_{zx}\| - \|s_{xx}\|\|p_{zz}\sigma_{zy}\|\right), \\
\overline{\rho}_{zz} &= \|p_{zz}\| + \frac{1}{D}\left((\|s_{yx}\|\|p_{zz}\sigma_{zy}\| - \|s_{yy}\|\|p_{zz}\sigma_{zx}\|)\right. \\
&\quad \times\|s_{xx}\rho_{xz} + s_{xy}\rho_{yz}\| \\
&\quad +(\|s_{xy}\|\|p_{zz}\sigma_{zx}\| - \|s_{xx}\|\|p_{zz}\sigma_{zy}\|) \\
&\quad \left.\times \|s_{yx}\rho_{xz} + s_{yy}\rho_{yz}\|\right), \qquad (22)
\end{aligned}$$

where we have introduced the denominator $D \equiv \|s_{xx}\|\|s_{yy}\| - \|s_{xy}\|\|s_{yx}\|$. Eqs. (20) and (22) are valid in the general case. We shall now give reduced expressions in some particular cases.

Without magnetization and for a cubic lattice, the symmetry imposes $\sigma^{l_1l_2}_{i\neq j} = \rho^{l_1l_2}_{i\neq j} = 0$. In addition, we have $s^{l_1l_2}_{ii} = \sigma^{l_1l_2}_{ii}$ for $i \in \{x, y\}$ and $p^{l_1l_2}_{zz} = \rho^{l_1l_2}_{zz}$ (see Eq. (15)). Thus, the effective conductivity and resistivity tensors (Eqs. (19) and (21), respectively) reduce to

$$\overline{\sigma} = \begin{pmatrix} \|\sigma_{xx}\| & 0 & 0 \\ 0 & \|\sigma_{yy}\| & 0 \\ 0 & 0 & \frac{1}{\|\rho_{zz}\|} \end{pmatrix}, \quad (23)$$

$$\overline{\rho} = \begin{pmatrix} \frac{1}{\|\sigma_{xx}\|} & 0 & 0 \\ 0 & \frac{1}{\|\sigma_{yy}\|} & 0 \\ 0 & 0 & \|\rho_{zz}\| \end{pmatrix}. \quad (24)$$

When the magnetization is along the x-direction, we have $\sigma^{l_1l_2}_{ix} = \sigma^{l_1l_2}_{xi} = \rho^{l_1l_2}_{ix} = \rho^{l_1l_2}_{xi} = 0$ for $i \in \{y, z\}$ (and similarly for s and p). They are no particular relations between $s^{l_1l_2}_{ij}$ and $\sigma^{l_1l_2}_{ij}$ or between $p^{l_1l_2}_{ij}$ and $\rho^{l_1l_2}_{ij}$. Thus, the effective conductivity and resistivity tensors are equals to

$$\overline{\sigma} = \begin{pmatrix} \|s_{xx}\| & 0 & 0 \\ 0 & \|s_{yy}\| - \frac{\|s_{yy}\rho_{yz}\|\|p_{zz}\sigma_{zy}\|}{\|p_{zz}\|} & -\frac{\|s_{yy}\rho_{yz}\|}{\|p_{zz}\|} \\ 0 & \frac{\|p_{zz}\sigma_{zy}\|}{\|p_{zz}\|} & \frac{1}{\|p_{zz}\|} \end{pmatrix}, \quad (25)$$

$$\overline{\rho} = \begin{pmatrix} \frac{1}{\|s_{xx}\|} & 0 & 0 \\ 0 & \frac{1}{\|s_{yy}\|} & \frac{\|s_{yy}\rho_{yz}\|}{\|s_{yy}\|} \\ 0 & -\frac{\|p_{zz}\sigma_{zy}\|}{\|s_{yy}\|} & \|p_{zz}\| - \frac{\|s_{yy}\rho_{yz}\|\|p_{zz}\sigma_{zy}\|}{\|s_{yy}\|} \end{pmatrix}, \quad (26)$$

where $D = \|s_{xx}\|\|s_{yy}\|$. The results with a magnetization along the y-direction can be obtained from Eqs. (25) and (26) by exchanging x and y indices.

When the magnetization is along the z-direction, we have $\sigma^{l_1l_2}_{iz} = \sigma^{l_1l_2}_{zi} = \rho^{l_1l_2}_{iz} = \rho^{l_1l_2}_{zi} = 0$ for $i \in \{x, y\}$. In addition, we have $s^{l_1l_2}_{ij} = \sigma^{l_1l_2}_{ij}$ for $\{i, j\} \in \{x, y\}$ and $p^{l_1l_2}_{zz} = \rho^{l_1l_2}_{zz}$ (see Eq. (15)). Thus, the effective conductivity and resistivity tensors are equals to

$$\overline{\sigma} = \begin{pmatrix} \|\sigma_{xx}\| & \|\sigma_{xy}\| & 0 \\ \|\sigma_{yx}\| & \|\sigma_{yy}\| & 0 \\ 0 & 0 & \frac{1}{\|\rho_{zz}\|} \end{pmatrix}, \quad (27)$$

$$\overline{\rho} = \begin{pmatrix} \frac{1}{D}\|\sigma_{yy}\| & -\frac{1}{D}\|\sigma_{xy}\| & 0 \\ -\frac{1}{D}\|\sigma_{yx}\| & \frac{1}{D}\|\sigma_{xx}\| & 0 \\ 0 & 0 & \|\rho_{zz}\| \end{pmatrix}, \quad (28)$$

where $D = \|\sigma_{xx}\|\|\sigma_{yy}\| - \|\sigma_{xy}\|\|\sigma_{yx}\|$. The fact that for CIP, the effective conductivity $\overline{\sigma}_{ij}$ (where $\{i, j\} \in \{x, y\}$) is simply equal to $\|\sigma_{ij}\| = \sum_{l_1,l_2} \sigma^{l_1,l_2}_{ij}/L$ and that for CPP, the effective conductivity $\overline{\sigma}_{zz}$ is equal to $1/\|\rho_{zz}\| = 1/(\sum_{l_1,l_2} \rho^{l_1,l_2}_{zz}/L)$ has been widely used in the literature[3,6,7]. Assuming the local limit, it is thus possible to model the multilayer as a network of resistors in series (in the CIP geometry) or in parallels (in the CPP geometry)[3,8].

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